# Nuclear structure functions and off-shell corrections

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## Abstract

The $n$-th moments of the nuclear structure function $F_2^A(x, Q^2)$ are analyzed using the off-shell kinematics appropriate to describe deep inelastic electron-nucleus scattering within the impulse approximation. It is shown that off-shell effects are sensitive to the form of both the nuclear spectral function and the nucleon structure function $F_2^N(x, Q^2)$, and can be as large as $\sim 10\%$ at $Q^2 \sim 2$ (GeV/c)$^2$.

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Nuclear effects in deep inelastic lepton-nucleus scattering (DIS) have been extensively studied, both experimentally and theoretically, over the last two decades (for a review see refs. [1,2]). The body of available data provides clearcut evidence that the nucleus cannot be simply described as a collection of nucleons on the mass shell. Proton knock-out experiments in which the outgoing particle is detected in coincidence with the scattered electron have clearly shown that the nuclear spectral function $P(\mathbf{p}, E)$, yielding the probability of finding a nucleon with momentum $\mathbf{p}$ and removal energy $E$ in the target nucleus, extends to very large values of $|\mathbf{p}|$ ($\gtrsim$ 800 MeV/c) and $E$ ($\gtrsim$ 200 MeV) [3]. Hence, the theoretical description of lepton scattering off fast off-shell nucleons must be regarded as a prerequisite for the understanding of nuclear DIS data.

A pioneering study of the electromagnetic response of a bound nucleon, focused on the intermediate energy domain, has been carried out in the 80's by de Forest [4]. More recent theoretical analyses of off-shell effects in nuclear DIS [5,6] have been stimulated by the availability of new experimental information on the $Q^2$-dependence of the moments of the nuclear structure function $F_2^A(x, Q^2)$, showing sizable deviations from the prediction of the simple convolution model [5].

In this paper we study the relevance of nuclear effects to the $Q^2$-dependence of the nuclear structure function $F_2^A(x, Q^2)$ and its moments using the appropriate off-shell kinematics and a realistic nuclear spectral function, resulting from *ab initio* microscopic many-body calculations [7].

Let us consider electron scattering off a nucleon carrying four momentum $p \equiv (p_0, \mathbf{p})$ with $p^2 \neq m^2$, $m$ being the nucleon mass. As it is well known (see, e.g., ref. [8]), the differential cross section for the inclusive process $e + N \to e' + X$, in which the hadronic final state is undetected, can be written in the form

$$\frac{d^2\sigma_N}{d\Omega dE_{e'}} = \frac{1}{\mathrm{F}(k,p)} \frac{\alpha^2}{Q^4} \frac{E_{e'}}{E_e} L^{\mu\nu} W_{\mu\nu} \ , \tag{1}$$

where $\alpha^2 = e^2/4\pi = 1/137$, $q^2 = \nu^2 - |\mathbf{q}|^2 = -Q^2$ is the squared four-momentum tranfer and $E_e$ and $E_{e'}$ are the initial and final electron energy, respectively. The function

$\mathrm{F}(k,p) = v_{rel} = |(\mathbf{k}/E_e) - (\mathbf{p}/p_0)|$, $k \equiv (E_e, \mathbf{k})$ being the four momentum carried by the incoming electron, describes the incident flux. In the case of ultrarelativistic electrons it takes the simple form $\mathrm{F}(k,p) = (kp)/(p_0 E_e)$. Note that for an isolated stationary nucleon $F \equiv 1$.

The electromagnetic structure of the electron and the nucleon is described by the tensors $L^{\mu\nu}$ and $W_{\mu\nu}$, respectively. Expanding the hadronic tensor in terms of the available relativistic invariants and taking into account current conservation, the contraction $L^{\mu\nu}W_{\mu\nu}$ can be readily written in terms of two nucleon structure functions $W_1$ and $W_2$:

$$L^{\mu\nu}W_{\mu\nu} = 2\left\{2\ W_1(kk') + \frac{W_2}{m^2}\left[2(kp)(k'p) - p^2(kk')\right]\right\} \ , \tag{2}$$

where $k' = k + q$ denotes the four momentum carried by the outgoing electron and $(kk') = 2E_e E_{e'} \sin^2(\theta/2)$, $\theta$ being the electron scattering angle. In the case of scattering off a free nucleon at rest $p^2 = m^2$, $(kp)(k'p) = m^2 E_e E_{e'}$ and substitution of eq.(2) into eq.(1) yields that standard result

$$\frac{d^2\sigma_N}{d\Omega dE_{e'}} = \sigma_M \left[W_2 + 2\ W_1 \tan^2\frac{\theta}{2}\right] \ , \tag{3}$$

where $\sigma_M = \alpha^2 \cos^2(\theta/2)/[4E_e^2 \sin^4(\theta/2)]$ is the Mott cross section and the nucleon structure functions depend upon $q^2$ and $(qp)$ only. In general, the inclusive electron-nucleon cross-section contains additional terms and can be cast in the form

$$\frac{d^2\sigma_N}{d\Omega dE_{e'}} = \frac{\sigma_M}{\mathrm{F}(k,p)} \left[W_2\,(1-\Delta) + 2\ W_1 \tan^2\frac{\theta}{2}\right] \ , \tag{4}$$

with

$$\Delta(k,k',p) = \left[1 - \frac{(kp)}{mE_e}\frac{(k'p)}{mE_{e'}}\right]\cos^{-2}\frac{\theta}{2} - \left(1 - \frac{p^2}{m^2}\right)\tan^2\frac{\theta}{2} \ , \tag{5}$$

and the structure functions $W_1$ and $W_2$ depend upon the three relativistic invariants $q^2$, $(qp)$ and $p^2$. For an isolated stationary nucleon $p^2 = m^2$, $\mathrm{F}(k,p) = 1$, $\Delta(k,k',p) = 0$ and eq.(3) is recovered.

The electron-nucleon cross section of eq.(4) can be used to obtain the inclusive electron-nucleus cross section within the impulse approximation from [9]:

$$\frac{d\sigma_A}{d\Omega dE_{e'}} = \int d^4p \left(\frac{m}{p_0}\right) P(p) \left[\mathrm{F}(k,p) \left(\frac{d^2\sigma_N}{d\Omega dE_{e'}}\right)\right] \tag{6}$$

where $P(p) \equiv P(|\mathbf{p}|, m - p_0)$ denotes the nuclear spectral function. The interpretation of the above equation is straightforward. The electron-nucleus cross section is written as the incoherent sum of the cross sections associated with scattering off individual nucleons, weighted with the nuclear spectral function and stripped of the flux factors, since the incident flux has to be defined with respect to the target, i.e. to the nucleus as a whole. Note that the nuclear cross section of eq.(6) can be recast in a form similar to eq.(3), with the nucleon structure functions $W_1$ and $W_2$ replaced by two nuclear structure functions $W_1^A$ and $W_2^A$.

Due to the fact that $P(p)$ is a steeply decreasing function of $|\mathbf{p}|$, the main contribution to the $|\mathbf{p}|$ integration involved in eq.(6) comes from the region of $\mathbf{p}^2$ small compared to $m^2$, where the use of a nonrelativistic nuclear spectral function is quite reasonable. Furthermore, as a first approximation, the functions $W_1$ and $W_2$, whose $p^2$-dependence is unknown, can be estimated assuming $W_{1,2}(q^2,(qp),p^2) \simeq W_{1,2}(q^2,(qp),m^2)$, and $W_{1,2}(q^2,(qp),m^2)$ can be extracted from electron-proton and electron-deuteron scattering data.

Using eqs.(3) and (6) the nuclear structure function $F_2^A = \nu W_2^A$ can be related to the nucleon structure function $F_2 = \nu W_2$ through

$$F_2^A(x,Q^2) = \int_x^A dz\ \phi_0(z,Q^2) F_2\left(\frac{x}{z},Q^2\right)\ , \tag{7}$$

where

$$\phi_0(z,Q^2) = z \int d^4p\ \left(\frac{m}{p_0}\right) P(p)\ \left[1 - \Delta(k,k',p)\right]\ \delta\left(z - \frac{(pq)}{(p_Aq)}\frac{M_A}{m}\right)\ , \tag{8}$$

$M_A$ being the mass of the target nucleus.

Note that setting $\Delta = 0$ eq.(7) reduces to the standard convolution formula (see, e.g., ref [10]):

$$\widetilde{F}_2^A(x,Q^2) = \int_x^A dz f_A(z,Q^2) F_2\left(\frac{x}{z},Q^2\right)\ , \tag{9}$$

with

$$f_A(z,Q^2) = z \int d^4p \left(\frac{m}{p_0}\right) P(p)\ \delta\left(z - \frac{(pq)}{(P_A q)}\frac{M_A}{m}\right) \ . \tag{10}$$

Let us now focus on the calculation of the $n$-th moment of the nuclear structure function, defined as

$$M_n^A(Q^2) = \int_0^A dx\ x^{n-2} F_2^A(x,Q^2) \ . \tag{11}$$

Using the convolution model result of eq.(9) and assuming the validity of the Bjorken limit, which amounts to disregard the $Q^2$-dependence of $f_A(z,Q^2)$ of eq.(10), eq.(11) can be rewritten in the simplified form

$$\widetilde{M}_n^A(Q^2) = \int_0^A dz \int_0^1 dy\ z^{n-1} f_A(z) y^{n-2} F_2\left(y,Q^2\right) \ , \tag{12}$$

leading to the factorized expression

$$\widetilde{M}_n^A(Q^2) = f_A^{(n+1)} M_n^N(Q^2) \ . \tag{13}$$

In the above equation $M_n^N(Q^2)$ is the n-th nucleon moment and

$$f_A^{(n+1)} = \int_0^A dz\ z^{n-1}\ f_A(z) \ , \tag{14}$$

whereas $f_A(z)$ denotes the $Q^2 \to \infty$ limit of $f_A(z,Q^2)$ [10].

Substitution of the nuclear structure function given by eqs.(7) and (8) into eq.(11) shows that the factorization property exhibited by the n-th moment evaluated within the convolution model breaks down when the Fermi motion and binding of the struck nucleon are properly taken into acount.

The breaking of factorization can be best observed studying the ratio between the nuclear and nucleon n-th moments, defined as

$$R_n(Q^2) = \frac{M_n^A(Q^2)}{M_n^N(Q^2)} \ . \tag{15}$$

According to the convolution model, $R_n(Q^2)$ is $Q^2$-independent, hence the study of its $Q^2$ dependence may provide useful information on the relevance of the factorization breaking terms arising from the inclusion of off-shell kinematics for the struck nucleon in the calculation of $F_2^A$ from eqs.(7)-(8).

The results of our numerical investigation of the effects of factorization breaking on the ratio $R_5(Q^2)$ are shown in fig.1. The solid curve has been obtained using $F_2^A(x, Q^2)$ given by eqs.(7) and (8) and the nuclear matter spectral function of ref. [7], whereas the data points, taken from ref. [5], have been extracted from the analysis of several experiments carried out at CERN [11] and SLAC [12,13] using $^{56}$Fe targets. It appears that the calculated $R_5(Q^2)$ decreases in the range $2 \lesssim Q^2 \lesssim 10$ (GeV/c)$^2$, while becoming almost constant at $Q^2 > 10$ (GeV/c)$^2$. The maximum deviation from the prediction of the convolution model, shown by the horizontal line in fig. 1, is $\sim 15\%$. Our results appreciably depend upon the form of the nucleon structure function $F_2(x, Q^2)$, particularly at $Q^2 < 10$ (GeV/c)$^2$. In this region we have employed the parametrization proposed in refs. [14,15], generally referred to as CKMT model, that provides a consistent formulation of the structure function at low $Q^2$ and gives a good description of the recent data from HERA [16–20]. According to the CKMT model, based on Regge theory, the nucleon structure function $F_2^N$ can be split into two parts: the singlet term, corresponding to the Pomeron contribution, and the nonsinglet term, corresponding to the secondary Reggeon. Both terms have the nonpertubative $Q^2$-dependence. At $Q^2 \geq 10$ (GeV/c)$^2$ we have used the so called GRV model, proposed in ref. [21], which reproduces the $Q^2$-evolution of $F_2(x, Q^2)$ quite well.

We have also used our approach to calculate the quantity

$$\tau_n^A(Q^2) = \left[\frac{M_n^A(Q^2)}{M_n^A(Q_0^2)}\right]^{-(1/d_n)} , \tag{16}$$

where $d_n = \gamma_n/2\beta_0$, $\gamma_n$ is the so called non singlet (NS) anomalous dimension and $\beta_0 = 11-(2/3)n_f$, $n_f$ being the number of flavors.

The solid line of fig. 2 shows $\tau_5^A(Q^2)$, obtained from eqs.(7)-(8) and (11), for $Q_0^2 = 12.5$ (GeV/c)$^2$. It exhibits a deviation from the $\log(Q^2/\Lambda^2)$ behavior predicted by leading order perturbative QCD and the factorization theorem. This deviation is a consequence of *both* higher twist corrrections, included in the CKMT parametrization of the nucleon structure function, *and* nuclear effects. In order to illustrate the relative importance of the two sources of non logarithmic $Q^2$-dependence of $\tau_5^A(Q^2)$, we also show, by the dashed line, the results obtained setting $\Delta = 0$ in eq.(8). Comparison between the solid and dashed lines shows that nuclear effects reach a maximum of $\sim 10$ % at $Q^2 = 2(GeV/c)^2$.

The results presented in figs. 1 and 2 can be related to each other noting that, defining the nucleon analog of $\tau_5^A(Q^2)$ as $\tau_5^N(Q^2) = [M_5^N(Q^2)/M_5^N(Q_0^2)]^{-1/d_5}$, we can write

$$\frac{\tau_5^N(Q^2)}{\tau_5^A(Q^2)} = \left[\frac{R_5(Q^2)}{R_5(Q_0^2)}\right]^{1/d_5} \quad . \tag{17}$$

Numerical calculations show that $\tau_5^N(Q^2)$ is very close to the quantity represented by the dashed line of fig. 2. Hence, to a very good approximation, the above equation provides a relationship between $R_5(Q^2)$ of fig. 1 and the ratio of the quantities represented by the dashed and solid lines of fig. 2. Since the experimental errors in $\tau_5^A(Q^2)$ are smaller than those associated with $R_5(Q^2)$ fig. 2 provides a better ilustration of our results. Note that we obtain similar results for all values of $n$ ($n$=3-8) for which data is available [5].

The $Q^2$-dependence of n-th moment ratios and the relevance of off-shell corrections has been recently discussed by Cothran *et al* [5]. In ref. [5] the structure function of an on-shell nucleon, $F_2^{ON}$ is written in terms of the relativistic invariant vertex function $\Phi(S)$ as

$$F_2^{ON}(x) = \frac{x^2}{1-x} \int \frac{d^2p_t}{2(2\pi)^3} \left[\frac{\Phi(S)}{x^2(S-m^2)}\right]^2 \quad , \tag{18}$$

where $S = p_t^2/(x(1-x)) + \lambda^2/(1-x) + \mu^2/x$, $\mu$ and $\lambda$ being the masses of the struck and spectator quarks respectively [5,8]. Applying Old-Fashioned-Perturbation-Theory to a system consisting of a constituent of mass $\mu$ and unit charge plus a neutral core of mass $\lambda$ one finds [22]

$$F_2(x) = \frac{x^2}{1-x} \int \frac{d^2p_t}{2(2\pi)^3} \int d\lambda^2 \left[\frac{\Phi_\lambda(x\widetilde{S})}{x\widetilde{S}}\right]^2 \quad , \tag{19}$$

where $\widetilde{S} = m^2 - S$ and $\Phi_\lambda(x\widetilde{S})/\widetilde{S}$ can be identified with the Fock space wave function in the infinite momentum frame. Comparison to eq.(19) shows that eq.(18) can be obtained carrying out an integration over the squared mass of the spectator system, $\lambda^2$, and is strictly applicable only in the $x \rightarrow 1$ limit (see, e.g., ref. [8]). Our numerical results show that the integrations involved in the calculations of the nuclear structure function (see eq.(7)) receive non negligible contributions from the region of small $(x/z)$ (for example, at $Q^2 = 2$ (GeV/c)$^2$ and $x$=0.7 more than 50 % of the integrated strength comes from the region $(x/z) \leq 0.7$). Hence, the results of the approach of ref. [5] should be taken with some caution.

In conclusion, we have shown that the use of off-shell kinematics, appropriate to describe lepton scattering off bound nucleons, in the calculation of the nuclear structure function $F_2^A(x, Q^2)$ leads to a sizable breakdown of factorization. The amount of the effect depends upon *both* the form of nuclear spectral function *and* the model of the nucleon structure function $F_2(x, Q^2)$.

Our approach is based on a standard nonrelativistic many-body treatment of nuclear dynamics, involving no adjustable parameters. Pionic and relativistic corrections may also play a role in this context, but unfortunately our formalism has not yet been developed to include their contributions in a consistent fashion.

We find that the the maximum deviation associated with off-shell effects is $\sim 10\%$ and occurs at $Q^2 \sim 2$ (GeV/c)$^2$, while at $Q^2 \geq 5$ (GeV/c)$^2$ the effect becomes negligibly small. Our calculations, based on a realistic spectral function and the CKMT model for the nucleon structure function, suggest that nuclear effects may be larger than previoulsy estimated [5], and that the extraction of higher twist corrections to $F_2(x, Q^2)$ from nuclear data may be questionable.

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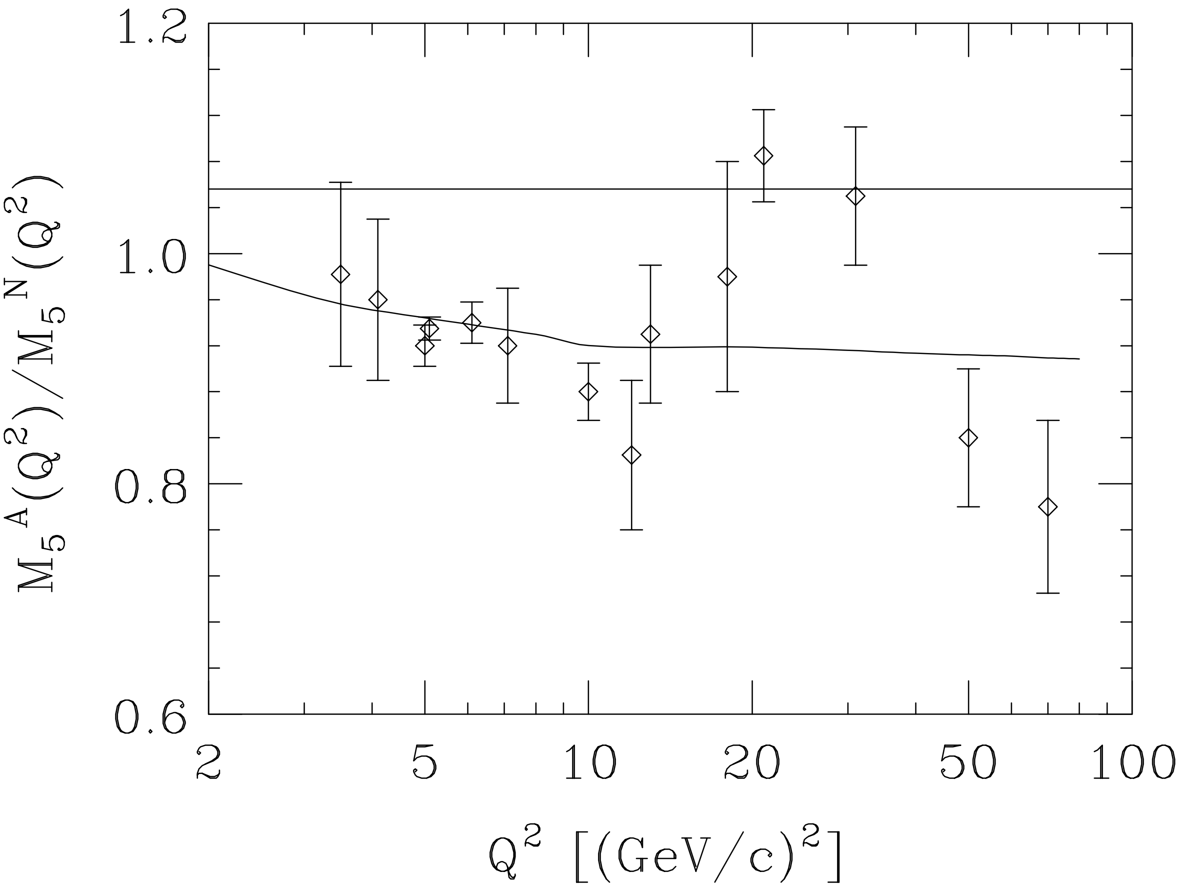
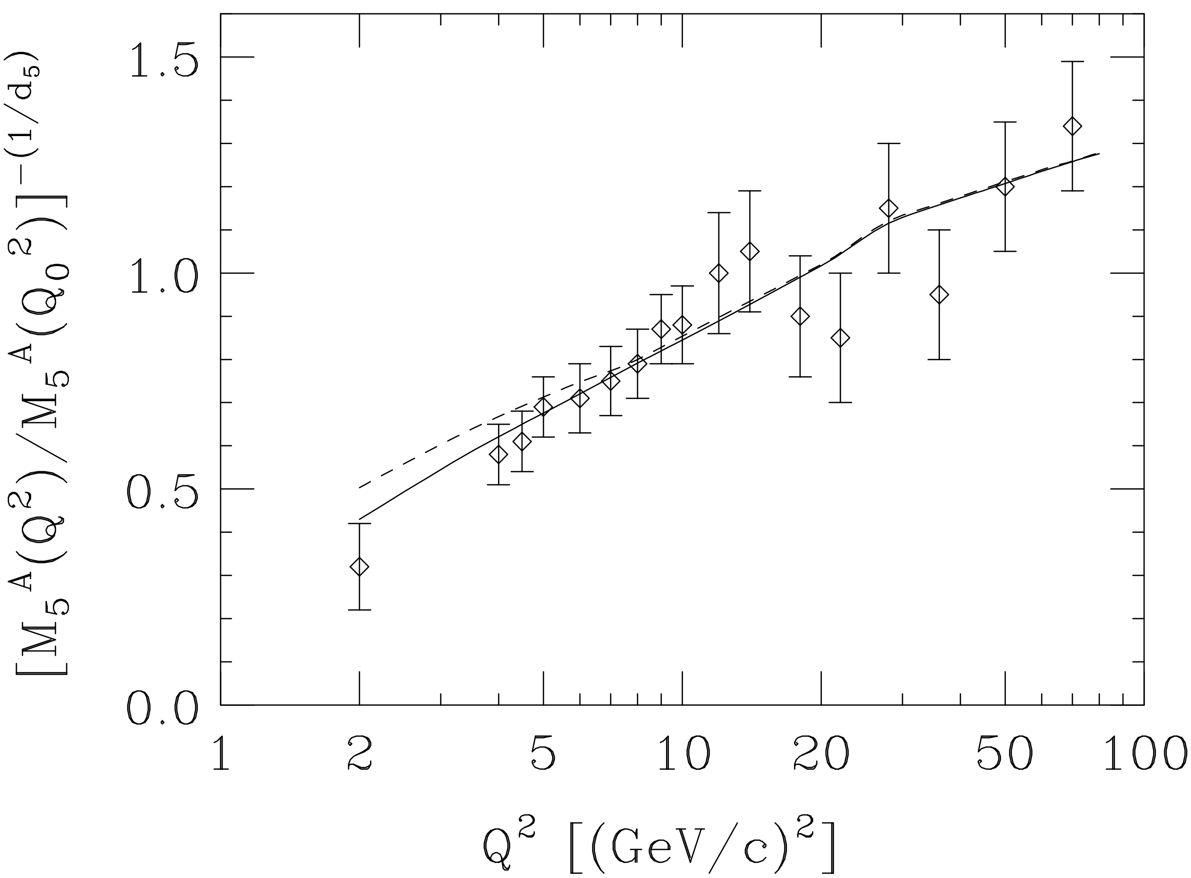
## FIGURES

FIG. 1. $Q^2$-dependence of $R_5(Q^2) = M_5^A(Q^2)/M_5^N(Q^2)$. The solid curve shows the results of our approach, whereas the horizontal line represents the prediction of the convolution model (eqs.(9) and (13)). The data is taken from ref. [5].

FIG. 2. $Q^2$-dependence of $[M_5^A(Q^2)/M_5^A(Q_0^2)]^{-1/d_5}$. The solid line corresponds to the full calculation, whereas the dashed line has been obtained neglecting off-shell corrections, i.e. setting $\Delta = 0$ in eq.(8).



1.2
1.0
0.8
0.6
2
5
10
20
50
100
$M_5^{\,A}(Q^2)/M_5^{\,N}(Q^2)$
$Q^2\ [(\mathrm{GeV}/c)^2]$



$[M_5{}^A(Q^2)/M_5{}^A(Q_0{}^2)]^{-(1/d_5)}$
0.0
0.5
1.0
1.5
1
2
5
10
20
50
100
$Q^2$ [(GeV/c)$^2$]